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A FEW SIMPLE OBSERVATIONS ON PION-CONDENSATION IN NUCLEI

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Abstract

We present a few simple observations on the occurrence of π-condensation in Nuclei, aimed at clarifying the nature of the π-condensation implied by the coherent nuclear π-N-Δ interaction, proposed in 1990 to explain the puzzling emergence of the Shell-Model. We show that such condensation is totally unrelated to the one proposed by A. B. Migdal at the beginning of '70, which can easily be shown not to occur at the normal nucleon density $\rho_N \simeq 0.17$ fm^{-3}.

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In 1990 a new approach to the dynamics of the Nucleus and of the Nuclear matter was proposed, based on an analogy between the *coherent* π-Nucleon QCD interaction in nuclear matter and the coherent QED interaction in ordinary condensed matter [1,2].

The rapidly growing research program aimed at elucidating the rôle of the coherent electrodynamical interactions among the constituents (atoms and molecules) of ordinary condensed matter, lent itself in a surprisingly natural way to a far reaching generalization to nuclear matter, that could finally clarify several points of Nuclear Physics that had remained mysterious at least to the natural philosopher, if not to the expert of the field. The mysteries we are referring to can all be essentially encapsulated in the following question: why is the Shell-Model (SM) such a good (approximate) description of the structure of the Nucleus ? A question that has puzzled the more thoughtful students of Nuclear Physics, since its proposal by Mayer and Jensen almost 50 years ago [3].

Let's analyze the origin and motivations of the puzzle which the SM poses to our physical intuition. Since the seminal ideas of Yukawa [4] nobody has ever put in doubt the notion that the nucleons of the Nucleus are held together by a "nucleostatic force", the Yukawa interaction, arising from the virtual exchange of π-mesons between pairs of nucleons. The finiteness of the π-mass, as well known, implies the exponential decay of such force as $e^{-m_\pi r}$, giving it the rather short range $R_{1\pi} = \frac{1}{m_\pi} \simeq 1.4$ fm. It is also well known that the basic π-exchange interaction has an important spin-isospin structure, which leads to repulsion instead of attraction in well defined spin-isospin channels. The same can be said of all other one-boson-exchanges which have, however, much smaller ranges [5]. Thus the only universal dynamical mechanism of attraction between nucleons, irrespective of their spin and isospin, responsible for the existence of highly complex Nuclei, has been identified in the 2π-exchange, involving the virtual transition to the $\Delta(1232)$ as well. The range of this kind of nuclear Van der Waals forces is thus $R_{2\pi} = \frac{1}{2m_\pi} \simeq 0.7$ fm, a remarkably small distance approximately equal to the radius of the nucleons.

Let us now take an assembly of nucleons and squeeze them in a volume so small that

their average mutual distance is comparable with $R_{2\pi}$[1]. Then what kind of equilibrium configuration can we expect for such system ?

A dense plasma with a stable neutralizing background bears a close resemblance to our nucleon system: in fact the short range pionic interaction is a rather accurate mock-up of the neutralizing background's interaction with the plasma which is Debye-screened at a distance comparable with the distances between charges. And the physics of such dense plasma is well known, resembling a kind of jelly where the charges, the seeds, oscillate around their equilibrium position with the "plasma frequency"[2]. Wouldn't it,then, be reasonable to expect such a "jellium" structure to accurately represent the dynamics of the Nucleus as well ? To such question the answer of the SM is a surprising, incontrovertible no. The nucleons of the Nucleus revolve around it in global orbits, much in the same way as the electrons whirl around the Nucleus in the Atom. Very strange, isn't it ?

Indeed, something that would have advised the nuclear physicists to look somewhere else in search of a physically realistic basis for the remarkable phenomenological success of the SM. From a historical point of view it is interesting to contemplate the initial skepticism and the later wonder of the leading nuclear physicists [6] when confronted with the simplicity

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[1] As a matter of fact for the actual average nucleon density $\rho_N \simeq 0.17$ fm^{-3} the intranucleon average distance $a_N \simeq \rho_N^{-1/3} \simeq 1.81$ fm turns out to be remarkably large, a fact that can be appreciated by computing the average of $\exp -\frac{|\vec{x}_1-\vec{x}_2|}{R_{2\pi}}$ for two nucleons with a gaussian density distribution of radius R_N which yields the small value 0.05, for $R_N \simeq 0.7$ fm, the nucleon's radius.

[2] In order to have some idea about the structure of such jelly, it is amusing to pursue in a crude way the analogy with a dense plasma. The plasma frequency is then [2]

$$\omega_p \simeq \frac{g}{m_N^{1/2}}\left(\frac{N}{V}\right)^{1/2} = \frac{g}{m_N^{1/2}}\left(\frac{1}{a_N}\right)^{3/2} \tag{1}$$

where $g \simeq 1$ yields at r $= R_{2\pi}$ the reasonable potential $V_{2\pi} \simeq 100$ MeV and $\omega_p \simeq 40$ MeV. The typical oscillation amplitude is then $\delta = \frac{1}{(m_N\omega_p)^{1/2}} \simeq 10^{-13}$ cm, a rather reasonable value, too.

and the effectiveness of the SM, skepticism and wonder that through habit the successive generations came to completely forget, deeply involved on one hand in the complicated calculations of nuclear structure, and on the other to check the "self-consistency" of the SM: a task that completely overlooked the fundamental question: why a dense plasma, governed in QED by a similar set of interactions has a dynamical behaviour which is completely different from that of the Nucleus ?

The 1990 paper, referred to above, finally succeeded in identifying a completely new interaction mechanism, whose basic structure was just what is needed to make sense, in a realistic way, of the SM and of other suitable aspects of nuclear dynamics[3]. In a nutshell the fundamental idea is that the Nucleon is just one level of the s-wave three non-strange quark system, whose excited state is $\Delta(1232)$, lying some 300 MeV above it. These two levels are strongly coupled to the π-field (itself a quark-antiquark system in s-wave), which induces the transitions $\pi + N \rightarrow \Delta(1232) \rightarrow \pi + N$ etc. The similarity of this physical system with the familiar Laser should not escape the attention of anybody. However, in the generally accepted view a system of such kind will "lase" if and only if it is "inverted", i.e. if through some suitable device - the pump - one brings a large number of atoms to the excited level. Furthermore it is important to place the system in a well-tuned optical cavity in order to prevent photons to leak out and be lost for the coherent laser evolution. If this were always true (it is certainly true in the operational conditions of the Lasers) the mechanism we are envisaging would be totally irrelevant, but it turns out that, contrary to what is generally believed, this is not always true. As demonstrated in 1973 by K. Hepp and E. Lieb [8], a system governed by the Dicke Hamiltonian [9] (such as the laser) above a certain density and below a certain temperature undergoes *spontaneously* a Superradiant Phase Transition (SPT) to a Laser-like state, where matter and a number of resonant modes of the e.m. field

[3]In [7] and in the Chapter 11 of the book [2] one can find a number of applications of this novel approach

interact coherently, oscillating in phase. And this without any need neither of pumps nor of cavities. This crucial and revolutionary result, which for mysterious reasons has had no impact on the Physics community, was rediscovered and generalized by one of us (G.P.) in 1987, and is the focal point of the book in Ref. [2]. Based upon it the Nucleus becomes a "*bona-fide*" Pionic Laser, whose two levels are just the Nucleon and the $\Delta(1232)$, and, as shown in Ref. [1], the couplings of both N(940) and $\Delta(1232)$ to the π-field are strong enough to meet the conditions for a SPT. In this way, through the coherent interaction with the π-field, which gets trapped in the region where the collective"N-Δ current" is localized, i.e. within the Nucleus, the assembly of Nucleons reaches a completely novel ground state, where N's and Δ's oscillate in phase and their "orbit" are not constrained to be localized, for the binding π-field is spread out throughout the Nucleus, and not peaked around the single Nucleons as envisaged by the short-range "nucleostatic" potential. As a matter of fact, as argued in Ref. [1], the SM just describes the ground state of a finite number of Fermions confined by their collective interactions within the nuclear volume. In a certain sense we may say that in the new approach the Nucleus owes its existence to a "condensation" of the π-field within the spatial extent of the Nucleus. But it is clear that such "condensate" is of a very peculiar type, characterized by its well defined phase relation with the N-Δ oscillations, and by the collective, coherent character of its interaction with the N-Δ system.

In spite of the remarkably successful phenomenology [7] that one can deduce from the precise quantum field theoretical formulation that has been expounded in Refs. [1,2], these ideas have found no interest nor resonance in the community of nuclear physics. The "mystery" of such a consistent neglect of both the conceptual difficulties of nucleostatic forces *vis-à-vis* the SM and the nuclear structure in general, and the satisfactory and theoretically compelling solution by the coherent nuclear interaction sketched above, has recently been lifted in the occasion of a review of our work demanded by a funding Agency. We have finally learnt that this approach has been foresaken by the community for it violates a well know result in Nuclear Physics, which goes back to the beginning of the 70's, and is associated mainly with the work of the Russian physicist A. B. Migdal [10]. According to this work,

which has been subsequently refined in many ways[4], π-condensation at the actual nuclear densities $\rho_N \simeq 0.17$ fm^{-3} is ruled out by a strong repulsion effects which push the critical density $\rho_c \simeq 3\rho_N$, way above what can be realized in a Nucleus.

It is the purpose of the last observation to clarify why the above argument is totally irrelevant for assessing the validity of the approach of the Coherent Nucleus. In simple terms, as described in Ref. [11], the problem of π-condensation is dealt with by analysing the propagator of a π-field $D(\omega, \vec{k}\,)$ in a gas of Nucleons of density ρ. The condition of condensation is then reduced to finding whether the inverse propagator (Π is the self energy function)

$$D^{-1}(\omega, \vec{k}\,) = \omega^2 - \vec{k}^{\,2} - m_\pi^2 - \Pi(\omega, \vec{k}\,) \tag{2}$$

has a zero for $\omega \leq 0$, identifying the critical density ρ_{crit} as that density for which the pole of $D(\omega, \vec{k}\,)$ is at $\omega = 0$, i.e.

$$D^{-1}(0, \vec{k}\,) = -\vec{k}^{\,2} - m_\pi^2 - \Pi(0, \vec{k}\,) = 0. \tag{3}$$

The negative, generally accepted, conclusion about π-condensation in ordinary Nuclei stems from the results of a calculation of the π-propagator which sums *incoherently* the contributions (particle-hole) of each of the Nucleons. In this way one obtain for the π-self energy

$$\Pi(\omega, \vec{k}\,) = -\frac{\vec{k}^{\,2}\chi_0(\omega, \vec{k}\,)}{1 + g'\chi_0(\omega, \vec{k}\,)} \tag{4}$$

where the susceptibility function $\chi_0(\omega, \vec{k}\,)$ receives contributions from both N(940) and Δ(1232) and is proportional to the nuclear density ρ, as implied by the *incoherent* sum. g' is the "correlation parameter", originating from short-range repulsion.

It should be now abundantly clear that the pion condensation that is predicted by the coherent π-N-Δ interaction is totally unrelated to the one familiar to the nuclear physicists, and that the impossibility of the latter cannot have any bearing on the likelihood of the

[4]For a simple but very clear account see the book by Ericsson and Weise [11]

former, which, besides its conceptual advantages,has on its side an impressive number of successes [1,2,7].

To conclude, whereas *incoherent* π-condensation is definitely ruled out by both theory and experiment, the coherent "superradiant" process that produces a *coherent* π-condensate appears not only solidly rooted in theory but also supported by experiments, beginning with the stunning effectiveness of the SM.

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